12-4 Multiplying Probabilities

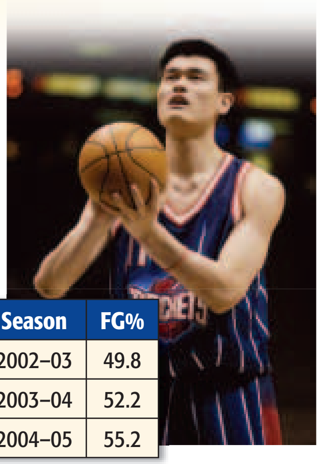
Main Ideas

- Find the probability of two independent events.
- Find the probability of two dependent events.

New Vocabulary

area diagram

GET READY for the Lesson



Yao Ming, of the Houston Rockets, has one of the best field-goal percentages in the National Basketball Association. The table shows the field-goal percentages for three years of his career. For any year, you can determine the probability that Yao will make two field goals in a row based on the probability of his making one field goal.

Season	FG%
2002–03	49.8
2003–04	52.2
2004–05	55.2

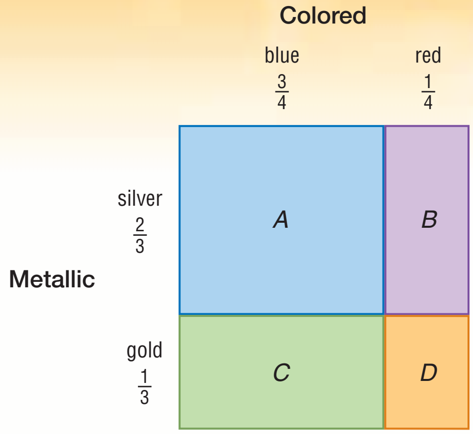
Source: nba.com

Probability of Independent Events In a situation with two events like shooting a field goal and then shooting another, you can find the probability of both events occurring if you know the probability of each event occurring. You can use an **area diagram** to model the probability of the two events occurring.

Algebra Lab

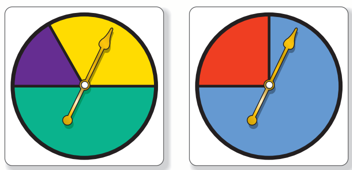
Area Diagrams

Suppose there are 1 red and 3 blue paper clips in one drawer and 1 gold and 2 silver paper clips in another drawer. The area diagram represents the probabilities of choosing one colored paper clip and one metallic paper clip if one of each is chosen at random. For example, rectangle A represents drawing 1 silver clip and 1 blue clip.



MODEL AND ANALYZE

1. Find the areas of rectangles A, B, C, and D. Explain what each represents.
2. Find the probability of choosing a red paper clip and a silver paper clip.
3. What are the length and width of the whole square? What is the area? Why does the area need to have this value?
4. Make an area diagram that represents the probability of each outcome if you spin each spinner once. Label the diagram and describe what the area of each rectangle represents.



In Exercise 4 of the lab, spinning one spinner has no effect on the second spinner. These events are independent.

KEY CONCEPT — *Probability of Two Independent Events*

If two events, *A* and *B*, are independent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B)$.

This formula can be applied to any number of independent events.

EXAMPLE 1 Two Independent Events

At a picnic, Julio reaches into an ice-filled cooler containing 8 regular soft drinks and 5 diet soft drinks. He removes a can, then decides he is not really thirsty, and puts it back. What is the probability that Julio and the next person to reach into the cooler both randomly select a regular soft drink?

> **Study Tip**
>
> **Alternative Method**
>
> You could use the Fundamental Counting Principle to find the number of successes and the number of total outcomes.
>
> both regular = $8 \cdot 8$ or 64
>
> total outcomes = $13 \cdot 13$ or 169
>
> So, $P(\text{both reg.}) = \frac{64}{169}$

Explore These events are independent since Julio replaced the can that he removed. The outcome of the second person's selection is not affected by Julio's selection.

Plan Since there are 13 cans, the probability of each person's getting a regular soft drink is $\frac{8}{13}$.

Solve

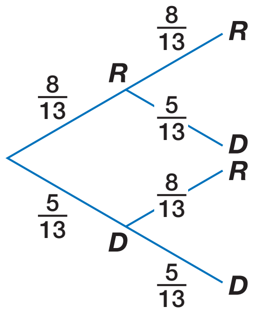
$P(\text{both regular}) = P(\text{regular}) \cdot P(\text{regular})$ — Probability of independent events

$= \frac{8}{13} \cdot \frac{8}{13}$ or $\frac{64}{169}$ — Substitute and multiply.

The probability that both people select a regular soft drink is $\frac{64}{169}$ or about 38%.

Check You can verify this result by making a tree diagram that includes probabilities. Let *R* stand for regular and *D* stand for diet.

$P(R, R) = \frac{8}{13} \cdot \frac{8}{13}$



CHECK Your Progress

1. At a promotional event, a radio station lets visitors spin a prize wheel. The wheel has 10 sectors of the same size for posters, 6 for T-shirts, and 2 for concert tickets. What is the probability that two consecutive visitors will win posters?

EXAMPLE 2 Three Independent Events

In a board game, three dice are rolled to determine the number of moves for the players. What is the probability that the first die shows a 6, the second die shows a 6, and the third die does not?

> **Study Tip**
>
> The **complement** of a set is the set of all objects that do *not* belong to the given set. For a six-sided die, showing a 6 is the complement of showing 1, 2, 3, 4, or 5.

Let *A* be the event that the first die shows a 6. → $P(A) = \frac{1}{6}$

Let *B* be the event that the second die shows a 6. → $P(B) = \frac{1}{6}$

Let *C* be the event that the third die does *not* show a 6. → $P(C) = \frac{5}{6}$

$P(A, B, \text{and } C) = P(A) \cdot P(B) \cdot P(C)$ Probability of independent events

$= \frac{1}{6} \cdot \frac{1}{6} \cdot \frac{5}{6}$ or $\frac{5}{216}$ Substitute and multiply.

The probability that the first and second dice show a 6 and the third die does not is $\frac{5}{216}$.

CHECK Your Progress

2. In a state lottery game, each of three cages contains 10 balls. The balls are each labeled with one of the digits 0–9. What is the probability that the first two balls drawn will be even and that the third will be prime?

Study Tip

Conditional Probability

The event of getting a regular soft drink the second time *given* that Julio got a regular soft drink the first time is called a *conditional probability*.

Probability of Dependent Events In Example 1, what is the probability that both people select a regular soft drink if Julio does not put his back in the cooler? In this case, the two events are dependent because the outcome of the first event affects the outcome of the second event.

First selection	**Second selection**
$P(\text{regular}) = \frac{8}{13}$	$P(\text{regular}) = \frac{7}{12}$

Notice that when Julio removes his can, there is not only one fewer regular soft drink but also one fewer drink in the cooler.

$P(\text{both regular}) = P(\text{regular}) \cdot P(\text{regular following regular})$

$= \frac{8}{13} \cdot \frac{7}{12}$ or $\frac{14}{39}$ Substitute and multiply.

The probability that both people select a regular soft drink is $\frac{14}{39}$ or about 36%.

KEY CONCEPT *Probability of Two Dependent Events*

If two events, A and B, are dependent, then the probability of both events occurring is $P(A \text{ and } B) = P(A) \cdot P(B \text{ following } A)$.

This formula can be extended to any number of dependent events.

EXAMPLE 3 Two Dependent Events

The host of a game show is drawing chips from a bag to determine the prizes for which contestants will play. Of the 10 chips in the bag, 6 show *television*, 3 show *vacation*, and 1 shows *car*. If the host draws the chips at random and does not replace them, find the probability that he draws a vacation, then a car.

Because the first chip is not replaced, the events are dependent. Let T represent a television, V a vacation, and C a car.

$P(V \text{ and } C) = P(V) \cdot P(C \text{ following } V)$ Dependent events

$= \frac{3}{10} \cdot \frac{1}{9}$ or $\frac{1}{30}$ After the first chip is drawn, there are 9 left.

The probability of a vacation and then a car is $\frac{1}{30}$ or about 3%.

CHECK Your Progress

3. Use the information above. What is the probability that the host draws two televisions?

EXAMPLE 4 Three Dependent Events

Three cards are drawn from a standard deck of cards without replacement. Find the probability of drawing a diamond, a club, and another diamond in that order.

Since the cards are not replaced, the events are dependent. Let D represent a diamond and C a club.

$$P(D, C, D) = P(D) \cdot P(C \text{ following } D) \cdot P(D \text{ following } D \text{ and } C)$$

$$= \frac{13}{52} \cdot \frac{13}{51} \cdot \frac{12}{50} \text{ or } \frac{13}{850}$$

If the first two cards are a diamond and a club, then 12 of the remaining cards are diamonds.

The probability is $\frac{13}{850}$ or about 1.5%.

CHECK Your Progress

4. Find the probability of drawing three cards of the same suit.

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CHECK Your Understanding

Example 1 (p. 704)

A die is rolled twice. Find each probability.

1. P(5, then 1)

2. P(two even numbers)

Examples 1, 3 (pp. 704, 705)

There are 8 action, 3 comedy, and 5 children's DVDs on a shelf. Suppose two DVDs are selected at random from the shelf. Find each probability.

3. P(2 action DVDs), if replacement occurs

4. P(2 action DVDs), if no replacement occurs

5. P(a comedy DVD, then a children's DVD), if no replacement occurs

Examples 2, 4 (pp. 704, 706)

Three cards are drawn from a standard deck of cards. Find each probability.

6. P(3 hearts), if replacement occurs

7. P(3 hearts), if no replacement occurs

Determine whether the events are *independent* or *dependent*. Then find the probability.

8. A black die and a white die are rolled. What is the probability that a 3 shows on the black die and a 5 shows on the white die?

9. Yana has 4 black socks, 6 blue socks, and 8 white socks in his drawer. If he selects three socks at random with no replacement, what is the probability that he will first select a blue sock, then a black sock, and then another blue sock?

Example 3 (p. 705)

Two cards are drawn from a standard deck of cards. Find each probability if no replacement occurs.

10. P(two hearts)

11. P(ace, then king)

Exercises

A die is rolled twice. Find each probability.

12. P(2, then 3)

13. P(no 6s)

14. P(two 4s)

15. P(1, then any number)

16. P(two of the same number)

17. P(two different numbers)

HOMEWORK HELP	
For Exercises	See Examples
12–20	1
21–29	3
30–35	1–4

The tiles *E, T, F, U, N, X,* and *P* of a word game are placed face down in the lid of the game. If two tiles are chosen at random, find each probability.

18. *P*(*E*, then *N*), if replacement occurs

19. *P*(2 consonants), if replacement occurs

20. *P*(*T*, then *D*), if replacement occurs

21. *P*(*X*, then *P*), if no replacement occurs

22. *P*(2 consonants), if no replacement occurs

23. *P*(selecting the same letter twice), if no replacement occurs

Anita scores well enough at a carnival game that she gets to randomly draw two prizes out of a prize bag. There are 6 purple T-shirts, 8 yellow T-shirts, and 5 T-shirts with a picture of a celebrity on them in the bag. Find each probability.

24. *P*(choosing 2 purple)

25. *P*(choosing 2 celebrity)

26. *P*(choosing a yellow, then a purple)

27. *P*(choosing a celebrity, then a yellow)

28. **ELECTIONS** Tami, Sonia, Malik, and Roger are the four candidates for Student Council president. If their names are placed in random order on the ballot, what is the probability that Malik's name will be first on the ballot followed by Sonia's name second?

29. **CHORES** The five children of the Blanchard family get weekly chores assigned to them at random. Their parents put pieces of paper with the names of the five children in a hat and draw them out. The order of the names pulled determines the order in which the children will be responsible for sorting laundry for the next five weeks. What is the probability that Jim will be responsible for the first week and Emily will be responsible for the fifth week?

Real-World Link

Three hamsters domesticated in 1930 are the ancestors of most of the hamsters sold as pets or used for research.

Source: www.ahc.umn.edu

Determine whether the events are *independent* or *dependent*. Then find the probability.

30. There are 3 miniature chocolate bars and 5 peanut butter cups in a candy dish. Judie chooses 2 of them at random. What is the probability that she chose 2 miniature chocolate bars?

31. A cage contains 3 white and 6 brown hamsters. Maggie randomly selects one, puts it back, and then randomly selects another. What is the probability that both selections were white?

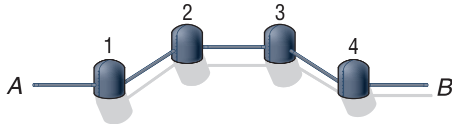
32. A bag contains 7 red, 4 blue, and 6 yellow marbles. If 3 marbles are selected in succession, what is the probability of selecting blue, then yellow, then red, if replacement occurs each time?

33. Jen's purse contains three \$1 bills, four \$5 bills, and two \$10 bills. If she selects three bills in succession, find the probability of selecting a \$10 bill, then a \$5 bill, and then a \$1 bill if the bills are not replaced.

34. What is the probability of getting heads each time if a coin is tossed 5 times?

35. When Ramon plays basketball, he makes an average of two out of every three foul shots he takes. What is the probability that he will make the next three foul shots in a row?

36. **UTILITIES** A city water system includes a sequence of 4 pumps as shown below. Water enters the system at point A, is pumped through the system by pumps at locations 1, 2, 3, and 4, and exits the system at point B.



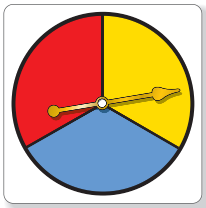
If the probability of failure for any one pump is $\frac{1}{100}$, what is the probability that water will flow all the way through the system from A to B?

Real-World Link

There are 2,598,960 different 5-card hands that can be drawn from a 52-card deck. Of these, only 5108 are hands in which all 5 cards are of the same suit.

37. **FISHING** Suppose a sport fisher has a 35% chance of catching a fish that he can keep each time he goes to a spot. What is the probability that he catches a fish the first 4 times he visits the spot but on the fifth visit he does not?

For Exercises 38–41, suppose you spin the spinner twice.



38. Sketch a tree diagram showing all of the possibilities. Use it to find the probability of spinning red and then blue.

39. Sketch an area diagram of the outcomes. Shade the region on your area diagram corresponding to getting the same color twice.

40. What is the probability that you get the same color on both spins?

41. If you got the same color twice, what is the probability that the color was red?

Find each probability if 13 cards are drawn from a standard deck of cards and no replacement occurs.

42. P(all hearts)

43. P(all red cards)

44. P(all one suit)

45. P(no kings)

For Exercises 46–48, use the following information.

A bag contains 10 marbles. In this problem, a *cycle* means that you draw a marble, record its color, and put it back.

46. You go through the cycle 10 times. If you do not record any black marbles, can you conclude that there are no black marbles in the bag?

47. Can you conclude that there are none if you repeat the cycle 50 times?

48. How many times do you have to repeat the cycle to be certain that there are no black marbles in the bag? Explain your reasoning.

EXTRA PRACTICE

See pages 917, 937.

Math Online

Self-Check Quiz at algebra2.com

H.O.T. Problems

49. **OPEN ENDED** Describe two real-life events that are dependent.

50. **FIND THE ERROR** Mario and Tamara are calculating the probability of getting a 4 and then a 2 if they roll a die twice. Who is correct? Explain your reasoning.

Mario

$P(4, \text{then } 2) = \frac{1}{6} \cdot \frac{1}{6}$

$= \frac{1}{36}$

Tamara

$P(4, \text{then } 2) = \frac{1}{6} \cdot \frac{1}{5}$

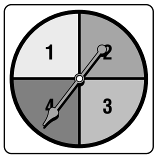
$= \frac{1}{30}$

51. **CHALLENGE** If one bulb in a string of holiday lights fails to work, the whole string will not light. If each bulb in a set has a 99.5% chance of working, what is the maximum number of lights that can be strung together with at least a 90% chance of the whole string lighting?

52. ***Writing in Math*** Use the information on page 703 to explain how probability applies to basketball. Explain how a value such as one of those in the table could be used to find the chances of Yao Ming making 0, 1, or 2 of 2 successive field goals, assuming the 2 field goals are independent, and a possible reason why 2 field goals might not be independent.

STANDARDIZED TEST PRACTICE

53. **ACT/SAT** The spinner is spun four times. What is the probability that the spinner lands on 2 each time?



A $\frac{1}{2}$

B $\frac{1}{4}$

C $\frac{1}{16}$

D $\frac{1}{256}$

54. **REVIEW** A coin is tossed and a die is rolled. What is the probability of a head and a 3?

F $\frac{1}{4}$

G $\frac{1}{8}$

H $\frac{1}{12}$

J $\frac{1}{24}$

Spiral Review

A gumball machine contains 7 red, 8 orange, 9 purple, 7 white, and 5 yellow gumballs. Tyson buys 3 gumballs. Find each probability, assuming that the machine dispenses the gumballs at random. (Lesson 12-3)

55. P(3 red)

56. P(2 white, 1 purple)

57. **PHOTOGRAPHY** A photographer is taking a picture of a bride and groom together with 6 attendants. How many ways can he arrange the 8 people in a row if the bride and groom stand in the middle? (Lesson 12-2)

Solve each equation. Check your solutions. (Lesson 9-3)

58. $\log_5 5 + \log_5 x = \log_5 30$

59. $\log_{16} c - 2 \log_{16} 3 = \log_{16} 4$

Given a polynomial and one of its factors, find the remaining factors of the polynomial. Some factors may not be binomials. (Lesson 6-7)

60. $x^3 - x^2 - 10x + 6;\ x + 3$

61. $x^3 - 7x^2 + 12x;\ x - 3$

GET READY for the Next Lesson

PREREQUISITE SKILL **Find each sum if $a = \frac{1}{2}$, $b = \frac{1}{6}$, $c = \frac{2}{3}$, and $d = \frac{3}{4}$.**

62. $a + b$

63. $b + c$

64. $a + d$

65. $b + d$

66. $c + a$

67. $c + d$